Written methods for division of whole numbers

The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for division which they know they can rely on when mental methods are not appropriate.

These notes show the stages in building up to long division through Years 4 to 6 - first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division - for example in 18 ÷ 3 = 6,the 18 is the dividend, the 3 is the divisor and the 6 is the quotient
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways
- recall multiplication and division facts to 10 × 10, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value
- know how to find a remainder working mentally - for example, find the remainder when 48 is divided by 5
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction
- estimate how many times one number divides into another - for example, how many sixes there are in 47, or how many 23s there are in 92
- multiply a two-digit number by a single-digit number mentally
- subtract numbers using the column method.

Method	Example
Stage 1: Mental division using partitioning	
Mental methods for dividing TU ÷ U can be based on partitioning and on the distributive law of division over addition. This allows a multiple of the divisor and the remaining number to be divided separately. The results are then added to find	One way to work out TU ÷ U mentally is to partition TU into a multiple of the divisor plus the remaining ones, then divide each part separately.

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the total quotient.

Many children can partition and multiply with confidence. But this is not the case for division. One reason for this may be that mental methods of division, stressing the correspondence to mental methods of multiplication, have not in the past been given enough attention.

Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.
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Informal recording in Year 4 for 84 ÷ 7 might be:
$\begin{array}{ccccl} & 84 & & & \\ 70 & + & 14 & & \\ \downarrow & & \downarrow & \div 7 & \\ 10 & + & 2 & = 12 & \end{array}$

In this example, using knowledge of multiples, the 84 is partitioned into 70 (the highest multiple of 7 that is also a multiple of 10 and less than 84) plus 14 and then each part is divided separately using the distributive law.

Another way to record is in a grid, with links to the grid method of multiplication.
$\begin{array}{|c|c|c|}\hline \times & & \\ \hline 7 & 70 & 14 \\ \hline \end{array} \rightarrow \begin{array}{|c|c|c|}\hline \times & 10 & 2 \\ \hline 7 & 70 & 14 \\ \hline \end{array} \quad 10 + 2 = 12$

As the mental method is recorded, ask: 'How many sevens in seventy?' and: 'How many sevens in fourteen?'

Also record mental division using partitioning:
$\begin{aligned} 64 \div 4 &= (40 + 24) \div 4 \\ &= (40 \div 4) + (24 \div 4) \\ &= 10 + 6 = 16 \end{aligned}$
$\begin{aligned} 87 \div 3 &= (60 + 27) \div 3 \\ &= (60 \div 3) + (27 \div 3) \\ &= 20 + 9 = 29 \end{aligned}$

Remainders after division can be recorded similarly.
$\begin{aligned} 96 \div 7 &= (70 + 26) \div 7 \\ &= (70 \div 7) + (26 \div 7) \\ &= 10 + 3 \text{ R } 5 = 13 \text{ R } 5 \end{aligned}$
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<tr><td colspan="2">Stage 2: Short division of TU ÷ U</td></tr>
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'Short' division of TU ÷ U can be introduced as a more compact recording of the mental method of partitioning.

Short division of two-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

For most children this will be at the end of Year 4 or the beginning of Year 5.

The accompanying patter is 'How many threes divide into 80 so that
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For 81 ÷ 3, the dividend of 81 is split into 60, the highest multiple of 3 that is also a multiple 10 and less than 81, to give 60 + 21. Each number is then divided by 3.
$\begin{aligned} 81 \div 3 &= (60 + 21) \div 3 \\ &= (60 \div 3) + (21 \div 3) \\ &= 20 + 7 \\ &= 27 \end{aligned}$

The short division method is recorded like this:
$\begin{array}{r} 20 + 7 \\ 3\overline{)60 + 21} \end{array}$

This is then shortened to:
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Method	Example
the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7.	$\begin{array}{r} 2\;7 \\ 3\overline{)8\;{}^{2}1} \end{array}$ The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3. In second it is written as a superscript. The 27 written above the line represents the answer: 20 + 7, or 2 tens and 7 ones.
Stage 3: 'Expanded' method for HTU ÷ U	
This method is based on subtracting multiples of the divisor from the number to be divided, the dividend. For TU ÷ U there is a link to the mental method. As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?' Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same. This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract. Chunking is useful for reminding children of the link between division and repeated subtraction. However, children need to recognise that chunking is inefficient if too many subtractions have to be carried out. Encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.	97 ÷ 9 $\begin{array}{rl} 9\overline{)97} & \\ -\underline{90} & 9\times10 \\ 7 & \\ \text{Answer:} & 10\text{ R }7 \end{array}$ $\begin{array}{rl} 6\overline{)196} & \\ -\underline{60} & 6\times10 \\ 136 & \\ -\underline{60} & 6\times10 \\ 76 & \\ -\underline{60} & 6\times10 \\ 16 & \\ -\underline{12} & 6\times\underline{2} \\ 4 & 32 \\ \text{Answer:} & 32\text{ R }4 \end{array}$
The key to the efficiency of chunking lies in the estimate that is made before the chunking starts. Estimating for HTU ÷ U involves multiplying the divisor by multiples of 10 to find the two multiples that	To find 196 ÷ 6, we start by multiplying 6 by 10, 20, 30, ... to find that 6 × 30 = 180 and 6 × 40 = 240. The multiples of 180 and 240 trap the number 196. This tells us that the answer to 196 ÷ 6 is between 30 and 40. Start the division by first subtracting 180, leaving 16, and then

Method	Example
'trap' the HTU dividend. Estimating has two purposes when doing a division: ○ to help to choose a starting point for the division; ○ to check the answer after the calculation. Children who have a secure knowledge of multiplication facts and place value should be able to move on quickly to the more efficient recording on the right.	subtracting the largest possible multiple of 6, which is 12, leaving 4. $\begin{array}{rrl} & 6\overline{)196} & \\ - & \underline{180} & 6\times 30 \\ & 16 & \\ - & \underline{12} & 6\times \underline{2} \\ & 4 & \quad 32 \\ \text{Answer:} & & \quad 32\,R\,4 \end{array}$ The quotient 32 (with a remainder of 4) lies between 30 and 40, as predicted.
Stage 4: Short division of HTU ÷ U	
'Short' division of HTU ÷ U can be introduced as an alternative, more compact recording. No chunking is involved since the links are to partitioning, not repeated subtraction. The accompanying pattern is 'How many threes in 290?' (the answer must be a multiple of 10). This gives 90threes or 270, with 20 remaining. We now ask: 'How many threes in 21?' which has the answer 7. Short division of a three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound. For most children this will be at the end of Year 5 or the beginning of Year 6.	For 291 ÷ 3, because 3 × 90 = 270 and 3 × 100 = 300, we use 270 and split the dividend of 291 into 270 + 21. Each part is then divided by 3. $\begin{aligned} 291 \div 3 &= (270 + 21) \div 3 \\ &= (270 \div 3) + (21 \div 3) \\ &= 90 + 7 \\ &= 97 \end{aligned}$ The short division method is recorded like this: $3\overline{)290+1} = 3\overset{90\ +\ 7}{\overline{)270+21}}$ This is then shortened to: $3\overset{9\ 7}{\overline{)2\ 9\ {}^{2}1}}$ The carry digit '2' represents the 2 tens that have been exchanged for 20 ones. In the first recording above it is written in front of the 1 to show that a total of 21 ones are to be divided by 3. The 97 written above the line represents the answer: 90 + 7,or 9 tens and 7 ones.
Stage 5: Long division	
The next step is to tackle HTU ÷ TU, which for most children will be in Year 6. The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with	How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

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'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 3 ones forming the answer are recorded above the line, as in the second recording.

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$$\begin{array}{rrl} & 24\overline{)\,560} & \\ 20 & -\underline{480} & 24\times 20 \\ & 80 & \\ 3 & \underline{72} & 24\times 3 \\ & 8 & \end{array}$$

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$\begin{array}{r} 23 \\ 24\overline{)\,560} \\ -\underline{480} \\ 80 \\ -\underline{72} \\ 8 \end{array}$$

Answer: 23 R 8

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