Written methods for subtraction of whole numbers

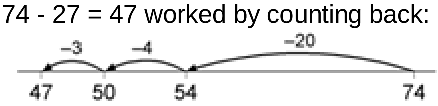
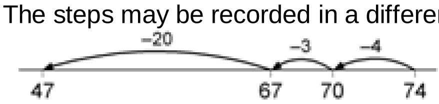
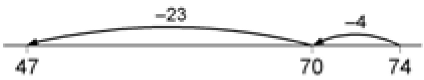
The aim is that children use mental methods when appropriate, but for calculations that they cannot do in their heads they use an efficient written method accurately and with confidence. Children are entitled to be taught and to acquire secure mental methods of calculation and one efficient written method of calculation for subtraction which they know they can rely on when mental methods are not appropriate.

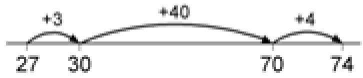
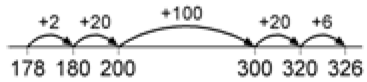
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20
- subtract multiples of 10 (such as 160 - 70) using the related subtraction fact, f16 - 7, and their knowledge of place value
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Method	Example
Stage 1: Using the empty number line	
The empty number line helps to record or explain the steps in mental subtraction. A calculation like 74 - 27 can be recorded by counting back 27 from 74 to reach 47. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. With practice, children will need to record less information and decide whether to count back or forward. It is useful to ask children whether counting up or back is the more efficient for calculations such as 57 - 12, 86 - 77 or 43 - 28. The notes below give more detail on the counting-up method using an empty number line.	Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. 15 - 7 = 8  74 - 27 = 47 worked by counting back:  The steps may be recorded in a different order:  or combined: 

Method	Example
The counting-up method	
The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ? = 74 mentally.	 74 − 27 3 → 30 40 → 70 4 → 74 47 Or:  74 − 27 3 → 30 44 → 74 47
With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ? = 200 and 200 + ? = 326 mentally. The most compact form of recording remains reasonably efficient.	 326 −178 2 → 180 20 → 200 100 → 300 26 → 326 148 Or:  326 −178 22 → 200 126 → 326 148
The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	 22.4 −17.8 0.2 → 18 4.0 → 22 0.4 → 22.4 4.6 Or:  22.4 −17.8 0.2 → 18 4.4 → 22.4 4.6
Stage 2: Partitioning	

<table>
<tr><th>Method</th><th>Example</th></tr>
<tr><td>Subtraction can be recorded using partitioning to write equivalent calculations that can be carried out mentally. For 74 - 27 this involves partitioning the 27 into 20 and 7, and then subtracting from 74 the 20 and the 4 in turn. Some children may need to partition the 74 into 70 + 4 or 60 + 14 to help them carry out the subtraction.</td><td>Subtraction can be recorded using partitioning:

74 - 27 = 74 - 20 - 7 = 54 - 7 = 47
74 - 27 = 70 + 4 - 20 - 7 = 60 + 14 - 20 - 7 = 40 + 7

This requires children to subtract a single-digit number or a multiple of 10 from a two-digit number mentally. The method of recording links to counting back on the number line.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<tr><td colspan="2">Stage 3: Expanded layout, leading to column method</td></tr>
<tr><td>Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.

This does not link directly to mental methods of counting back or up but parallels the partitioning method for addition. It also relies on secure mental skills.

The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and with partitioning.</td><td>Partitioned numbers are then written under one another:

Example: 74 - 27

$\begin{array}{r} 70+4 \\ -\ 20+7 \\ \hline \end{array}$ $\quad \begin{array}{r} \overset{60}{\cancel{70}}+\overset{14}{\cancel{4}} \\ -\ 20+7 \\ \hline 40+7 \end{array}$ $\quad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{14}{\cancel{4}} \\ -\ 2\ 7 \\ \hline 4\ 7 \end{array}$

Example: 741 - 367

$\begin{array}{r} 700+40+1 \\ -\ 300+60+7 \\ \hline \end{array}$ $\quad \begin{array}{r} \overset{600}{\cancel{700}}+\overset{130}{\cancel{40}}+\overset{11}{\cancel{1}} \\ -\ 300+60+7 \\ \hline 300+70+4 \end{array}$ $\quad \begin{array}{r} \overset{6}{\cancel{7}}\ \overset{13}{\cancel{4}}\ \overset{11}{\cancel{1}} \\ -\ 3\ 6\ 7 \\ \hline 3\ 7\ 4 \end{array}$</td></tr>
<tr><td colspan="2">The expanded method for three-digit numbers</td></tr>
<tr><td></td><td>Example: 563 − 241, no adjustment or decomposition needed

Expanded method
$\begin{array}{r} 500+60+3 \\ -\ 200+40+1 \\ \hline 300+20+2 \end{array}$

leading to
$\begin{array}{r} 563 \\ -\ 241 \\ \hline 322 \end{array}$

Start by subtracting the ones, then the tens, then the hundreds. Refer to subtracting the tens, for example, by saying 'sixty take</td></tr>
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<tr><th>Method</th><th>Example</th></tr>
<tr><td></td><td>

away forty', not 'six take away four'.

Example: 563 − 271, adjustment from the hundreds to the tens, or partitioning the hundreds

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+1 \\ \hline \end{array} \qquad \begin{array}{r} 400+160+3 \\ -\ 200+\ 70+1 \\ \hline 200+90+2 \end{array} \qquad \begin{array}{r} \overset{400}{\cancel{500}}+\overset{160}{\cancel{60}}+3 \\ -\ 200+70+1 \\ \hline 200+90+2 \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{16}{\cancel{6}}\,3 \\ -\ 2\,7\,1 \\ \hline 2\,9\,2 \end{array}$$

Begin by reading aloud the number from which we are subtracting: 'five hundred and sixty-three'. Then discuss the hundreds, tens and ones components of the number, and how 500 + 60 can be partitioned into 400 + 160. The subtraction of the tens becomes '160 minus 70', an application of subtraction of multiples of ten.

Example: 563 − 278, adjustment from the hundreds to the tens and the tens to the ones

$$\begin{array}{r} 500+60+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+150+13 \\ -\ 200+\ 70+\ 8 \\ \hline 200+\ 80+\ 5 \end{array} \qquad \begin{array}{r} \overset{400}{\overset{\cancel{500}}{\cancel{500}}}+\overset{150}{\overset{\cancel{50}}{\cancel{60}}}+\overset{13}{\overset{13}{\cancel{3}}} \\ -\ 200+70+8 \\ \hline 200+80+5 \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{15}{\cancel{6}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7\,8 \\ \hline 2\,8\,5 \end{array}$$

Here both the tens and the ones digits to be subtracted are bigger than both the tens and the ones digits you are subtracting from. Discuss how 60 + 3 is partitioned into 50 + 13, and then how 500 + 50 can be partitioned into 400 + 150, and how this helps when subtracting.

Example: 503 − 278, dealing with zeros when adjusting

$$\begin{array}{r} 500+\ 0+3 \\ -\ 200+70+8 \\ \hline \end{array} \qquad \begin{array}{r} 400+90+13 \\ -\ 200+70+\ 8 \\ \hline 200+20+\ 5 \end{array} \qquad \begin{array}{r} \overset{400}{\overset{\cancel{400}}{\cancel{500}}}+\overset{90}{\overset{\cancel{100}}{\cancel{0}}}+\overset{13}{\overset{3}{\cancel{3}}} \\ -\ 200+70+8 \\ \hline 200+20+5 \end{array}$$

$$\begin{array}{r} \overset{4}{\cancel{5}}\,\overset{9}{\cancel{0}}\,\overset{13}{\cancel{3}} \\ -\ 2\,7\,8 \\ \hline 2\,2\,5 \end{array}$$

Here 0 acts as a place holder for the tens. The adjustment has to be done in two stages. First the 500 + 0 is partitioned into 400 + 100 and then the 100 + 3 is partitioned into 90 + 13.

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